Abstracts

Quantum Topology and Hyperbolic Geometry Conference
Nha Trang, May 13–17, 2013

Trees and Wheels and Balloons and Hoops

Dror Bar-Natan (University of Toronto, Canada)

(Wednesday, 11:10)

Balloons are two-dimensional spheres. Hoops are one dimensional loops. Knotted Balloons and Hoops (KBH) in 4-space behave much like the first and second fundamental groups of a topological space – hoops can be composed like in π_1, balloons like in π_2, and hoops "act" on balloons as π_1 acts on π_2. We will observe that ordinary knots and tangles in 3-space map into KBH in 4-space and become amalgams of both balloons and hoops. We give an ansatz for a tree and wheel (that is, free-Lie and cyclic word)-valued invariant ζ of KBHs in terms of the said compositions and action and we explain its relationship with finite type invariants. We speculate that ζ is a complete evaluation of the BF topological quantum field theory in 4D, though we are not sure what that means. We show that a certain "reduction and repackaging" of ζ is an "ultimate Alexander invariant" that contains the Alexander polynomial (multivariable, if you wish), has extremely good composition properties, is evaluated in a topologically meaningful way, and is least-wasteful in a computational sense. If you believe in categorification, that's a wonderful playground.

Towards a Categorification of Quantum 3-Manifold Invariants

Anna Beliakova (University of Zurich)

(Wednesday, 10:10)

Witten-Reshetikhin-Turaev invariants of any homology 3-sphere at all roots of unity are dominated by a certain generating function – called a unified invariant, which takes its values in the Habiro ring. This ring is a cyclotomic completion of the polynomial ring in one variable with integral coefficients. In the talk we provide evidence to the fact that the unified invariants are more natural objects for categorification than the original invariants. A categorification program for unified invariants is based on a categorification of the universal $sl(2)$ link invariant. For a knot this invariant belongs to the center of quantum $sl(2)$. Together with K. Habiro we recently made a crucial step towards a categorification of the universal R-matrix. We constructed an unbounded bicomplex which belongs to the Drinfeld center of the Khovanov-Lauda 2-category, whose Euler characteristic is the ribbon element of quantum $sl(2)$.

Non Semisimple TQFTs

Christian Blanchet (Universite Paris Diderot-Paris 7)

(Thursday, 14:40)

Coostantino, Geer and Patureau have defined a series of 3-manifolds invariants based on nilpotent representations of quantum sl(2) at a root of unity. The corresponding category is non semisimple, and the usual modular category framework does not apply. They get a series of invariants, indexed by an integer $N \geq 2$, defined for 3-manifolds with colored graph and cohomology class

with coefficients in C/NZ. We show that these CGP invariants extend to TQFTs with interesting features. For the smallest root of unity i.e. $N = 2$, the construction produces a TQFT for a version of Reidemeister torsion.

Knot Invariants and Topological Recursion

Gaetan Borot

(Thursday, 15:40)

Towards the Classification of Quasi-Alternating Montesinos Links

Abhijit Champanerkar (College of Staten Island, CUNY)

(Monday, 14:00)

Quasi-alternating links are a generalization of alternating links from a knot homological perspective. They are homologically thin for both Khovanov homology and knot Floer homology. Recent work of Greene and my joint work with Kofman resulted in the classification of quasi-alternating pretzel links in terms of their integer tassel parameters. Replacing tassels by rational tangles generalizes pretzel links to Montesinos links. In this talk I will establish conditions on the rational parameters of a Montesinos link to be quasi-alternating. Using recent results on left-orderable groups and Heegaard Floer L-spaces, I will also establish conditions on the rational parameters of a Montesinos link to be non quasi-alternating. Finally I will discuss some open problems and conjectures about quasi-alternating links. This is joint work with Philip Ording.

Knot State Asymptotics (2)

Laurent Charles (Universite Pierre et Marie Curie, Paris 6)

(Thursday, 11:10)

In a joint work with J. Marché, we proved the Witten asymptotic conjecture for the manifolds obtained by surgery on the figure eight knot. Our approach is based on a semi-classical analysis of q-difference equations. We showed that the figure eight knot state behaves like a WKB solution of a Schrödinger equation. I will explain in my talk the meaning of this last assertion.

Flags, Triangulations, and Quantization

Tudor Dimofte (Institute for Advanced Study)

(Thursday, 9:00)

I will introduce the concept of framed flat connections on framed 3-manifolds. These are ordinary flat $PGL(K)$ connections together with an extra choice of invariant flag on distinguished parts of the boundary. The framing allows flat connections to be "localized" within a triangulation of a 3-manifold, so that they can be described with local coordinates that generalize the complex dihedral angles of Thurston in hyperbolic geometry. (These local coordinates are closely related to recent work of Garoufalidis-Goerner-Zickert.) More interestingly, the framing allows the moduli space of flat connections in a 3-manifold to be described as a Lagrangian submanifold in the moduli space of connections on the boundary. This then ties in beautifully with higher Teichmuller theory in two dimensions, where 3-manifolds represent cobordisms between different surfaces, and Lagrangian

submanifolds become Lagrangian correspondences. (Joint work with M. Gabella, D. Gaiotto, and A. Goncharov.)

The Jones Polynomial and Surfaces Far from Fibers

Dave Futer (Temple University)

(Tuesday, 11:10)

Continuing the theme of the previous talk, we will explore the relations between (colored) Jones polynomials and the topology and geometry of incompressible surfaces. Under mild hypotheses on a link diagram $D(K)$, we prove that coefficients of the Jones and colored Jones polynomials determine whether a particular surface is a fiber in the complement of K. Furthermore, when these coefficients are large, the surface is "far from being a fiber", which implies that the link complement has large hyperbolic volume. This is joint work with Effie Kalfagianni and Jessica Purcell.

Left-Orderability and 3-Manifold Groups

Cameron Gordon (University of Texas, Austin)

(Friday, 15:40)

We will discuss evidence for the conjecture that a prime rational homology 3-sphere is an L-space if and only if its fundmanental group is not left-orderable. This is joint work with Steve Boyer and Liam Watson.

Kirby Calculus for Null-Homologous Framed Links in 3-Manifolds

Kazuo Habiro (RIMS, Kyoto University)

(Monday, 15:40)

Kirby's calculus of framed links gives a necessary and sufficient condition for two framed links in the 3-sphere to have homeomorphic results of surgery. This result is used in the construction of 3-manifold invariants such as the Witten-Reshetikhin-Turaev invariants. Several extensions and refinements of Kirby's calculus for various classes of framed links in 3-manifolds have been developed. In this talk, we will consider framed links L in a 3-manifold M such that each component of L is null-homologous in M. The result involves a new move on framed links, which is related to the IHX relation in the theory of finite type invariants. This is joint work with Tamara Widmer.

Incompressible Surfaces and Jones Polynomials

Effie Kalfagianni (Michigan State University, East Lansing)

(Tuesday, 10:10)

Under a mild diagrammatic hypothesis we develop a setting for deriving relations between Jones polynomials and the topology and geometry of incompressible surfaces in knot complements: In particular the boundary slope of certain incompressible surfaces in a knot complement is determined by the growth of the degree of the colored Jones polynomial of the knot (as predicted by the Slope Conjecture). For hyperbolic knots, we show that the geometric type of these surfaces in the Thurston trichotomy is also determined by certain coefficients of the colored Jones polynomial of the knot. This is joint work with Dave Futer and Jessica Purcell.

Asymptotics of Quantum 3-Manifold Invariants

Joanna Kania-Bartoszynska (National Science Foundation)

(Tuesday, 14:40)

We will discuss possible extensions of 3-manifold invariants away from roots of unity. Given a knot in a 3-sphere we consider the 3-manifold which is the double of the knot complement. We show that a natural extension of the Witten-Reshetikhin-Turaev invariant of such manifolds to the value of the complex parameter equal -1 is not a manifold invariant. We conjecture an asymptotic formula for those invariants. This is joint work with Charles Frohman, University of Iowa.

Quantum Teichmller Theory and TQFT

Rinat Kashaev (University of Geneva)

(Monday, 11:10)

Quantum Teichmller theory leads to a three-dimensional TQFT of novel type which we call Teichmller TQFT. Faddeevs quantum dilogarithm plays the fundamental role in this theory. I will describe the present status of the Teichmller TQFT and discuss its relation to hyperbolic geometry in the semiclassical limit. This is a joint work with Jrgen Ellegaard Andersen.

Seifert Volumes and Dilogarithm Identities

Vu T. Khoi (Hanoi Institute of Mathematics)

(Thursday, 14:00)

In this talk, we show how to obtain new dilogarithm identities by computing the Seifert volume of manifolds obtained by Dehn surgery on knots in two different ways.

Twisted Alexander Polynomials and Incompressible Surfaces Given by Culler-Shalen Theory

Takahiro Kitayama (University of Tokyo, Japan)

(Thursday, 16:20)

The leading coefficients of twisted Alexander polynomials of a 3-manifold induce a regular function on the $SL(2, C)$-character variety. Dunfield, Friedl and Jackson conjectured that for a knot if an ideal point gives Seifert surfaces by Culler-Shalen theory, then the function has a finite value there. We give a similar sufficient condition for a general 3-manifold, which implies a partial affirmative answer to their conjecture.

Geometrically Maximal Knots

Ilya Kofman (College of Staten Island, CUNY)

(Monday, 14:40)

X.-S. Lin conjectured that weaving knots asymptotically maximize the hyperbolic volume per crossing. We give precise lower volume bounds for weaving knots that prove Lin's conjecture. This is joint work with Abhijit Champanerkar and Jessica Purcell.

Quantum Symmetry in Homological Representations of Braid Groups and Applications

Toshitake Kohno (Tokyo University)
(Tuesday, 9:00)

Homological representations of braid groups are defined as the action of homeomorphisms of a punctured disk on the homology of an abelian covering of its configuration space. These representations were extensively studied by Krammer and Bigelow. In this talk we show that specializations of the homological representations of braid groups are equivalent to the monodromy of the KZ equation with values in the space of null vectors in the tensor product of Verma modules when the parameters are generic. To prove this we use the description of the solution of the KZ equation due to Schechtman and Varchenko. We also discuss the case of special parameters related to conformal field theory and describe the KZ connection as a Gauss-Manin connection. We will give some applications of this construction. First, we describe the image and the kernel of the action of braid groups and mapping class groups on the space of conformal blocks. We give how combinatorial structures of braid groups such as dual Garside structures are recovered from homological representations of braid groups. Finally, we describe recent work in progress concerning the categorification of the above construction.

Getting Knot Invariants from Representation Theory via Howe Duality
Aaron Lauda (University of Southern California)
(Wednesday, 9:00)

It is a well understood story that one can extract link invariants associated to simple Lie algebras. These invariants are called Reshetikhin-Turaev invariants and the famous Jones polynomial is the simplest example. Kauffman showed that the Jones polynomial could be described very simply by replacing crossings in a knot diagram by various smoothings. In this talk we will explain Cautis-Kamnitzer-Licata's simple new approach to understanding these invariants using basic representation theory and the quantum Weyl group action. Their approach is based on a version of Howe duality for exterior algebras called skew-Howe duality. Even the graphical (or skein theory) description of these invariants can be recovered in an elementary way from this data. The advantage of this approach is that it suggests a 'categorification' where knot homology theories arise in an elementary way from higher representation theory and the structure of categorified quantum groups.

Knot State Asymptotics (1)
Julien Marché (Ecole Polytechnique)
(Thursday, 10:10)

This talk is the first in a series of two talks, joint with L. Charles. I will explain the strategy of our proof of the Witten asymptotic conjecture for the surgeries on the figure eight knot. Then, I will discuss which problems one has to solve to extend it to more general knots.

TQFT and Modular Representations of Mapping Class Groups
Gregor Masbaum (Institut de Mathematiques de Jussieu, Paris)
(Monday, 9:00)

This talk will be about modular representations in finite characteristic of mapping class groups of surfaces. These representations come from the theory of Integral $SO(3)$ Topological Quantum

Field Theory. I will discuss joint work with Reid in which we use these representations to answer a question of Hamenstaedt about finite quotients of the mapping class group. I will also present Verlinde-like dimension formulas for the irreducible factors of these representations in the case of equal characteristic.

Milnor Invariants for Primes and Maass Wave Forms

Masanori Morishita (Kyushu University)

(Wednesday, 8:20)

I give an analytic expression for Milnor triple invariants for prime numbers in terms of the Fourier coefficients of Maass wave forms. It was suggested by the analogy, due to M. Kapranov, between TQFT and Langlands correspondence.

Generators and Relations for the Representation Theory of Quantum $SL(n)$

Scott Morrison (Mathematical Sciences Institute, Australia)

(Tuesday, 14:00)

Skew Howe duality says that $SL(n)$ and $SL(m)$ acting on $\wedge^\bullet \mathbb{C}^n \otimes \mathbb{C}^m$ generate each other's commutant. Using this, we can give a generators mod relations presentation of the representation theory of $SL(n)$ (or its quantum analogues), with objects the exterior powers of the standard representation. We can read off the relations amongst intertwiners between m-fold products of exterior powers as the relations in a certain quotient (depending on n) of $U\mathfrak{gl}(m)$.

How Much Does the Colored Jones Polynomial of a Knot Know About Representations of the Fundamental Group to $SL(2;C)$?

Hitoshi Murakami (Tokyo Institute of Technology)

(Friday, 11:10)

The volume conjecture says that the colored Jones polynomial of a knot would know the volume of the knot complement. It is also conjectured that the colored Jones polynomial would also know the volume and the Reidemeister torsion associated with a representation of the fundamental group of the knot complement to $SL(2;C)$. In this talk I will give a survey of this kind of refinement of the volume conjecture.

Longitudes in SL_2-Representations of Link Groups and Milnor-Witt K_2-Groups of Fields

Takefumi Nosaka (RIMS, Kyoto University)

(Friday, 14:00)

I propose a natural K_2-value of an $SL_2(F)$-representations of link groups, by using longitudes of the links. Furthermore I observed the non-triviality, and compute the values for some hyperbolic link groups.

Invariants of 3-Manifolds via the Mapping Class Group and Planar Graphs

Michael Polyak (Technion, Israel)

(Tuesday, 15:40)

Given a word in the mapping class group we construct a planar graph with weights on edges and vertices. Perturbative invariants of a 3-manifold with the corresponding Heegard splitting are then calculated by a simple counting of certain subgraphs. We also discuss an alternative new definition of finite type invariants of 3-manifolds and skein relations for such invariants.

Khovanov Homology of B-Adequate Links has a Tail

Lev Rozansky (University of North Carolina, Chapel Hill)

(Monday, 10:10)

Complete Integrability of Character Varieties

Adam Sikora (SUNY Buffalo)

(Friday, 14:40)

By the work of Witten and Anderson, Witten-Reshetikhin-Turaev TQFTs arise from the the quantization of the spaces of flat connections on surfaces (i.e. their character varieties). The geometric quantization (with real polarization) of the $SU(2)$-character varieties was achieved by Jeffrey and Weitsman based on the fact that these character varieties are completely integrable. We prove complete integrability of G-character varieties for groups G of rank 2 and discuss higher rank groups. We also discuss the application of these results to geometric quantization a la Jeffrey-Weitsman.

A Generating Function for MOY Graphs

Roland van der Veen (University of California, Berkeley)

(Friday, 10:10)

MOY graphs are trivalent graphs that were used by Murakami, Ohtsuki and Yamada (MOY) to study the sl_N quantum knot invariants and the colored HOMFLY polynomial. These graphs may be regarded as a higher rank generalization of the Kauffman bracket approach to the Jones polynomial. We will show how to obtain an explicit formula for the generating function of all the colorings of a fixed graph. The generating function leads to new recursions for colored HOMFLY polynomials and raises some interesting geometric questions. Time permitting similar results for other quantum groups will be mentioned (joint work with Stavros Garoufalidis).

Thurston's Gluing Equations for $PGL(n, C)$

Christian Zickert (University of Maryland)

(Friday, 9:00)

Thurston's gluing equations are polynomial equations invented by Thurston to explicitly compute hyperbolic structures or, more generally, representations in $PGL(2, C)$. This is done via so called shape coordinates. We generalize the shape coordinates to obtain a parametrization of representations in $PGL(n, C)$. We give applications to quantum topology, and discuss an intriguing duality between the shape coordinates and the Ptolemy coordinates of Garoufalidis-Thurston-Zickert. The shape coordinates and Ptolemy coordinates can be viewed as 3-dimensional analogues of the X- and A-coordinates on higher Teichmuller spaces due to Fock and Goncharov.